CS 61A Summer 2026

Sequences, Trees

Discussion 4: July 7, 2026

Lists

Some of you already know list operations that we haven't covered yet, such as `append`. Don't use those today. All you need are list literals (e.g., `[1, 2, 3]`), item selection (e.g., `s[0]`), list addition (e.g., `[1] + [2, 3]`), `len` (e.g., `len(s)`), and slicing (e.g., `s[1:]`). Use those! There will be plenty of time for other list operations when we introduce them later this week.

The most important thing to remember about lists is that a non-empty list `s` can be split into its first element `s[0]` and the rest of the list `s[1:]`.

```
>>> s = [2, 3, 6, 4]
>>> s[0]
2
>>> s[1:]
[3, 6, 4]
```

A list comprehension describes the elements in a list and evaluates to a new list containing those elements.

There are two forms:

```
[<expression> for <element> in <sequence>]
[<expression> for <element> in <sequence> if <conditional>]
```

Here's an example that starts with `[1, 2, 3, 4]`, picks out the even elements `2` and `4` using `if i % 2 == 0`, then squares each of these using `i*i`. The purpose of `for i` is to give a name to each element in `[1, 2, 3, 4]`.

```
>>> [i*i for i in [1, 2, 3, 4] if i % 2 == 0]
[4, 16]
```

This list comprehension evaluates to a list of:

- The value of `i*i`
- For each element `i` in the sequence `[1, 2, 3, 4]`
- For which `i % 2 == 0`

In other words, this list comprehension will create a new list that contains the square of every even element of the original list `[1, 2, 3, 4]`.

We can also rewrite a list comprehension as an equivalent `for` statement, such as for the example above:

```
>>> result = []
>>> for i in [1, 2, 3, 4]:
...     if i % 2 == 0:
...         result = result + [i*i]
>>> result
[4, 16]
```

Q1: Even weighted

Write a function that takes a list `s` and returns a new list that keeps only the even-indexed elements of `s` and multiplies them by their corresponding index. First approach this problem with a normal `for` loop (without list comprehension).

```
def even_weighted_loop(s):
    """
    >>> x = [1, 2, 3, 4, 5, 6]
    >>> even_weighted_loop(x)
    [0, 6, 20]
    """
    result = []
    for i in range(len(s)):
        if i % 2 == 0:
            result = result + [i * s[i]]
    return result
```

Now that you've done it with a `for` loop, try it with a list comprehension!

```
def even_weighted_comprehension(s):
    """
    >>> x = [1, 2, 3, 4, 5, 6]
    >>> even_weighted_comprehension(x)
    [0, 6, 20]
    """
    return [i * s[i] for i in range(len(s)) if i % 2 == 0]
```

The key point to note is that instead of iterating over each element in the list, we must instead iterate over the indices of the list. Otherwise, there's no way to tell if we should keep a given element.

Dictionaries

A dictionary is a Python data structure that maps *keys* to *values*. For a dictionary `d`:

- Each key maps to exactly one value, which you can access with `d[<key>]`.
- If you try `d[<key>]` but `<key>` is not a valid key in `d`, you'll get an error.
- You can use `<key> in d` to test (`True` or `False`) whether a key is in `d`.
- `d.get(<key>, <default>)` will return the value that `<key>` maps to in `d`, or `<default>` if `<key>` is not a key in `d`.
- The sequence of keys or values or key-value pairs can be accessed using `d.keys()` or `d.values()` or `d.items()`, respectively.
- Keys can be of any immutable type (like strings, numbers, and tuples) but not mutable types (like lists).
- You can use a `d` in a `for` loop (such as `for k in d`). Doing so will iterate through the keys in `d`.

Q2: Happy Givers

In a certain discussion section, some people exchange gifts for the holiday season. We call two people **happy givers** if they give gifts to each other. Implement a function `happy_givers`, which takes in a `gifts` dictionary that maps people in the section to the person they gave a gift to. `happy_givers` outputs a list of all the happy givers in the section. The order of the list does not matter.

Note that if someone received but did not give a gift, they will not appear in the `gifts` dictionary as a key. (They'll appear only as a value.) You can assume that no one gives themself a gift.

Once you've found a solution, as a challenge, attempt to implement a solution in one line using a list comprehension.

Optional: Imagine an alternate case where the dictionary where each person is a key, and its value is a list of all the people they gave a gift to. Attempt to implement a solution to find a list of all happy givers with this configuration.

```
def happy_givers(gifts):
    """
    >>> gift_recipients = {
    ...     "Alice": "Eve", # Alice gave a gift to Eve
    ...     "Bob": "Finn",
    ...     "Christina": "Alice",
    ...     "David": "Gina", # Gina is not a key because she didn't give anyone a gift
    ...     "Eve": "Alice",
    ...     "Finn": "Bob",
    ... }
    >>> list(sorted(happy_givers(gift_recipients))) # Order does not matter
    ['Alice', 'Bob', 'Eve', 'Finn']
    """
    output = []
    for giver in gifts:
        recipient = gifts[giver]
        if recipient in gifts and gifts[recipient] == giver:
            output = output + [giver]
    return output
    # Alternate solution using list comprehension
    # return [giver for giver in gifts if gifts[giver] in gifts and gifts[gifts[giver]]
    == giver]

    #Optional problem solution
    #output = []
    #for giver in gifts:
    #   for recipient in gifts[giver]:
    #       if (recipient in gifts) and (giver in gifts[recipient]) and (giver not in
    output):
    #           output = output + [giver]
    #return output
```

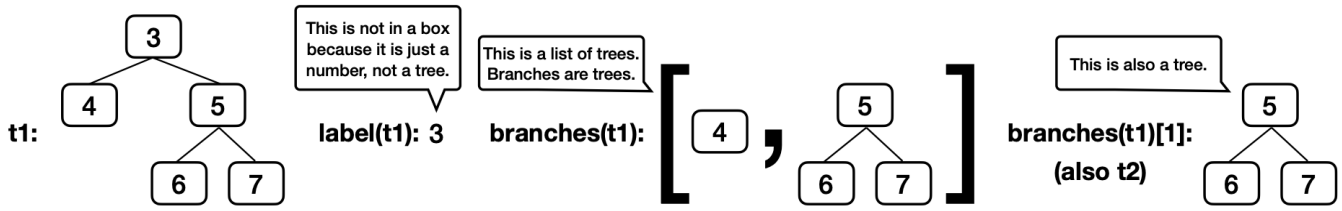
Trees

For a tree `t`:

- Its root label can be any value, and `label(t)` returns it.
- Its branches are trees, and `branches(t)` returns a list of branches.
- An identical tree can be constructed with `tree(label(t), branches(t))`.
- You can call functions that take trees as arguments, such as `is_leaf(t)`.
- That's how you work with trees. No `t == x` or `t[0]` or `x in t` or `list(t)`, etc.
- There's no way to change a tree (that doesn't violate an abstraction barrier).

Here's an example tree `t1`, for which its branch `branches(t1)[1]` is `t2`.

```
t2 = tree(5, [tree(6), tree(7)])
t1 = tree(3, [tree(4), t2])
```



Example Tree

A path is a sequence of trees in which each is the parent of the next.

```
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch), 'branches must be trees'
    return [label] + list(branches)

def label(tree):
    return tree[0]

def branches(tree):
    return tree[1:]

def is_leaf(tree):
    return not branches(tree)

def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
```

Q3: Warm Up

What value is bound to `result`?

```
result = label(min(branches(max([t1, t2], key=label)), key=label))
```

Solution

6: `max([t1, t2], key=label)` evaluates to the `t2` tree because its label 5 is larger than `t1`'s label 3. Among `t2`'s branches (which are leaves), the left one labeled 6 has a smaller label.

How convoluted!

Here's a quick refresher on how `key` functions work with `max` and `min`,

`max(s, key=f)` returns the item `x` in `s` for which `f(x)` is largest.

```
>>> s = [-3, -5, -4, -1, -2]
>>> max(s)
-1
>>> max(s, key=abs)
-5
>>> max([abs(x) for x in s])
5
```

Therefore, `max([t1, t2], key=label)` returns the tree with the largest label, in this case `t2`.

In case you're wondering, this expression does not violate an abstraction barrier. `[t1, t2]` and `branches(t)` are both lists (not trees), and so it's fine to call `min` and `max` on them.

Q4: Has Path

Implement `has_path`, which takes a tree `t` and a list `p`. It returns whether there is a path from the root of `t` with labels `p`. For example, `t1` has a path from its root with labels `[3, 5, 6]` but not `[3, 4, 6]` or `[5, 6]`.

Important: Before trying to implement this function, discuss these questions from lecture about the recursive call of a tree processing function: - What small initial choice can I make (such as which branch to explore)? - What recursive call should I make for each option? - How can I combine the results of those recursive calls? - What type of values do they return? - What do those return values mean?

```
def has_path(t, p):
    """Return whether tree t has a path from the root with labels p.

    >>> t2 = tree(5, [tree(6), tree(7)])
    >>> t1 = tree(3, [tree(4), t2])
    >>> has_path(t1, [5, 6])        # This path is not from the root of t1
    False
    >>> has_path(t2, [5, 6])        # This path is from the root of t2
    True
    >>> has_path(t1, [3, 5])        # This path does not go to a leaf, but that's ok
    True
    >>> has_path(t1, [3, 5, 6])     # This path goes to a leaf
    True
    >>> has_path(t1, [3, 4, 5, 6])  # There is no path with these labels
    False
    """
    if p == [label(t)]:
        return True
    elif label(t) != p[0]:
        return False
    else:
        return any([has_path(b, p[1:]) for b in branches(t)])

        for b in branches(t):
            if has_path(b, p[1:]):
                return True
        return False
```

Q5: Find Path

Implement `find_path`, which takes a tree `t` with unique labels and a value `x`. It returns a list containing the labels of the nodes along a path from the root of `t` to a node labeled `x`.

If `x` is not a label in `t`, return `None`. Assume that the labels of `t` are unique.

```
def find_path(t, x):
    """
    >>> t2 = tree(5, [tree(6), tree(7)])
    >>> t1 = tree(3, [tree(4), t2])
    >>> find_path(t1, 5)
    [3, 5]
    >>> find_path(t1, 4)
    [3, 4]
    >>> find_path(t1, 6)
    [3, 5, 6]
    >>> find_path(t2, 6)
    [5, 6]
    >>> print(find_path(t1, 2))
    None
    """
    if label(t) == x:
        return [label(t)]
    for b in branches(t):
        path = find_path(b, x)
        if path:
            return [label(t)] + path
    return None
```

The base case return value `[label(t)]` creates a one-element list of the labels along a path that starts at the root of `t` and also ends there, since the root is labeled `x`.

The assignment `path = find_path(b, x)` allows the return value of this recursive call to be used twice: once to check if it's `None` (which is a false value) and again to build a longer list.

The expression `[label(t)] + path` for a tree `t` and list `path` creates a longer list that starts with the label of `t` and continues with the elements of `path`.

Q6: Pruning Leaves

Implement `prune_leaves`, which takes a tree `t` and a tuple of values `vals`. It returns a version of `t` with all its **leaves** whose labels are in `vals` removed. Do not remove non-leaf nodes and do not remove leaves that do not match any of the items in `vals`. Return `None` if pruning the tree results in there being no nodes left in the tree.

def prune_leaves(t, vals): """Return a version of t (a new tree) with all leaves that have a label that appears in vals removed. Return None if the entire tree is pruned away.

```
>>> t = tree(2)
>>> print(prune_leaves(t, (1, 2)))
None
>>> numbers = tree(1, [tree(2), tree(3, [tree(4), tree(5)]), tree(6, [tree(7)])])
>>> print_tree(numbers)
1
  2
  3
    4
    5
  6
    7
>>> print_tree(prune_leaves(numbers, (3, 4, 6, 7)))
1
  2
  3
    5
  6
"""
if is_leaf(t) and (label(t) in vals):
  return None
new_branches = []
for b in branches(t):
  new_branch = prune_leaves(b, vals)
  if new_branch:
    new_branches.append(new_branch)
return tree(label(t), new_branches)
```